SOLVING REVERSE TRANSIENT-HEAT-CONDUCTION PROBLEMS BY ELECTRICAL SIMULATION

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Devices are described which make it possible to adapt existing models for a continuous-mode solution of both linear and nonlinear reverse transient-heat-conduction problems with variable boundary conditions.

In analyzing transient thermal process in turbomachinery, for example, it often becomes necessary to solve not only forward but also reverse transient-heat-conduction problems with variable boundary conditions.

According to [1], a reverse transient problem is one where the solution yields the coefficients in the boundary conditions, the latter being stated in a known form. The given quantities in this case are those describing the geometry of the body, the thermophysical properties of its material, and the transient temperature distribution at various points in the body including points on its surface.

Since analytical solutions to reverse problems are available only for a few most simple cases, hence in engineering practice such problems are solved essentially by the Liebmann method with RC networks. The procedure here is rather laborious, because for each instant of time the answer is sought by iteration with a manual resetting of the boundary conditions.

Forward transient-heat-conduction problems are nowadays solved widely by the use of electrical models – RC networks – in a continuous-mode computation process. Such networks are not suited for solving reverse problems, however, and this limits their application.

We propose here some device by means of which passive RC networks can be adapted for the solution of reverse transient-heat-conduction problems with variable boundary conditions, in a linear as well as in a nonlinear situation. The common feature of these devices is that they are designed on the principles of electronic simulation. As has been shown in [2, 3], the use of electronic devices with passive models does considerably extend the range of solvable problems.

The analog computer shown schematically in Fig. 1a will calculate the surface density of thermal flux $q_S(\tau)$ which ensures a given transient temperature distribution in a test object, i.e., will solve on an RC network the reverse problem with variable boundary conditions of the second kind:

$$q_s(\tau) = -\lambda \left(\frac{\partial T}{\partial n} \right)_s. \tag{1}$$

This device, like those which will be described here subsequently, represents a closed-loop automatic control system for the test object simulated by an RC network. The input quantity to this system is a voltage $U_S(\tau)$ proportional to the surface temperature of the body $T_S(\tau)$, which varies with time according to some given law, and the output quantity from the system is a current $I_T(\tau)$ fed to the terminal point of the model. As is well known, the current in this model is the electrical analog of the thermal flux density.

A function converter (FC_1) is used for shaping the voltage $U_S(\tau)$. This voltage is compared to the voltage at the terminal point of the model $U_T(\tau)$ and the mismatch signal

$$U_\varepsilon(\tau) = U_s(\tau) - U_T(\tau)$$

is applied to the input of the amplifier (DCA) which has a high gain K.

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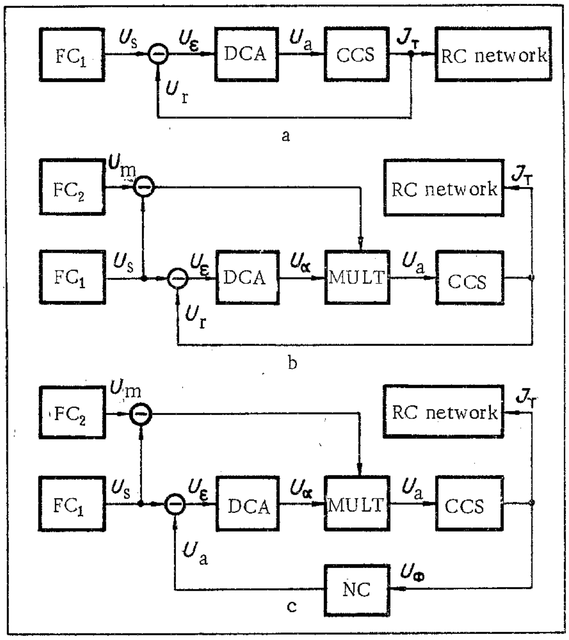

Fig. 1. Schematic block diagram of the device: a) for solving reverse problems with variable boundary conditions of the second kind; b) for solving reverse problems with variable boundary conditions of the third kind; c) for solving nonlinear reverse problems.

The voltage

$$U_a(\tau) = KU_\varepsilon(\tau)$$

from the DCA output proceeds to the input of the controlled current stabilizer (CCS), where it is converted into a current

$$I_T(\tau) = K_i U_a(\tau).$$

Inasmuch as the system is a closed one with a high gain $K \gg 1$, so the voltage $U_r(\tau)$, which is determined by the magnitude of current $I_T(\tau)$, will be such that at every instant of time the system $U_\varepsilon(\tau) \to 0$. Since also $U_r(\tau)$, hence current $I_T(\tau)$ will naturally be proportional to $q_s(\tau)$, i.e.,

$$I_T(\tau) = K_T q_s(\tau),$$

where $K_T = K_t/K_R$.

It is easier to measure voltage than current in the model and, therefore, the magnitude of current $I_T(\tau)$ can be indicated in terms of voltage $U_a(\tau)$, from which one may then proceed to evaluate $q_s(\tau)$:

$$q_s(\tau) = \frac{I_T(\tau)}{K_T} = \frac{K_i K_R}{K_t} U_a(\tau).$$

If the $q_s(\tau)$ level during heating of the body is maintained on the basis of boundary conditions of the third kind

$$\alpha(\tau)\,[T_m(\tau) - T_s(\tau)] = -\lambda\left(\frac{\partial T}{\partial n}\right)_s, \tag{2}$$

then further computation will yield the rate of heat transfer $\alpha(\tau)$ at the body surface:

$$\alpha(\tau) = \frac{q_s(\tau)}{T_m(\tau) - T_s(\tau)}. \tag{3}$$

The law according to which the temperature of the medium varies with time, i.e., the function $T_m(\tau)$ is assumed to be known here.

In order to cut out the intermediate calculations involved in determining $\alpha(\tau)$, it is worthwhile to solve reverse problems with variable boundary conditions of the third kind by means of a device shown schematically in Fig. 1b. This device, while solving Eq. (2) in implicit form, makes it possible to determine $\alpha(\tau)$ and $q_s(\tau)$ directly. For this purpose, a multiplicator (MULT) and another function converter (FC_2) are added so that a voltage $U_m(\tau)$ proportional to $T_m(\tau)$ will appear.

As has been noted already, voltage $U_a(\tau)$ at the input to the CCS channel is proportional to $q_s(\tau)$. At the same time, this voltage is also (see Fig. 1b):

$$U_a(\tau) = K_a U_\alpha(\tau)\,[U_m(\tau) - U_s(\tau)]. \tag{4}$$

It follows from expression (4) that voltage $U_\alpha(\tau)$ at the multiplicator input is the electrical analog of the heat-transfer rate $\alpha(\tau)$, i.e., that

$$U_\alpha(\tau) = K_\alpha\,\alpha(\tau),$$

where $K_\alpha = 1/K_i K_a K_R$.

The devices just described make it possible, by means of RC networks, to solve continuously reverse transient-heat-conduction problems in a linear situation, if the thermophysical properties of the material λ, c, γ are independent of the temperature.

We will show that the dependence of these thermophysical properties on the temperature can be accounted for in the solution of reverse problems, i.e., that a reverse problem can be solved with such a model in the nonlinear case too.

TABLE 1. Results of Solving a Reverse Problem for an Asymmetrically Heated Infinitely Large Plate at ($\bar{\theta}_{m_1} = T_{m_1}/T_{m_2} = 0.8$; $\bar{\theta}_{m_2} = 1$; $\bar{\theta}_0 = T_0/T_{m_2} = 0.2$; $T_{m_2} = 600°C$ = const)

Fo	Starting data		Results of electrical simulation						Analytical calculation			
	$\bar{\theta}_{S_1}$	$\bar{\theta}_{S_2}$	$\bar{\theta}_{S_1}$	$\bar{\theta}_{S_2}$	Ki_1	Ki_2	Bi_1	Bi_2	Ki_1	Ki_2	Bi_1	Bi_2
0	0,2	0,2	0,2	0,2	0,372	1,08	0,495	1,08	0,376	1,0	0,5	1,0
0,1	0,311	0,478	0,311	0,478	0,380	0,995	0,620	1,52	0,392	0,964	0,64	1,542
0,2	0,394	0,616	0,395	0,600	0,390	0,973	0,770	1,94	0,392	0,954	0,772	1,99
0,3	0,490	0,708	0,484	0,700	0,342	0,828	0,865	2,28	0,344	0,859	0,888	2,352
0,4	0,579	0,779	0,577	0,770	0,286	0,754	0,983	2,62	0,274	0,745	0,995	2,652
0,5	0,650	0,831	0,648	0,831	0,204	0,618	1,07	2,87	0,205	0,612	1,09	2,897
0,6	0,707	0,870	0,710	0,868	0,131	0,498	1,16	3,02	0,137	0,489	1,176	3,008
0,7	0,751	0,898	0,746	0,900	0,081	0,403	1,20	3,22	0,074	0,415	1,255	3,26
0,8	0,783	0,919	0,785	0,915	0,025	0,358	1,34	3,37	0,028	0,342	1,327	3,384
0,9	0,806	0,935	0,795	0,932	0,008	0,300	1,40	3,52	−0,0104	0,288	1,39	3,505
1,0	0,822	0,948	0,822	0,948	−0,042	0,235	1,52	3,61	−0,041	0,233	1,448	3,595
∞	0,857	0,971	0,857	0,971	−0,142	0,142	2,0	4,0	−0,142	0,145	2,0	4,0

The nonlinear equation of transient heat conduction

$$\frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[\lambda(T)\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[\lambda(T)\frac{\partial T}{\partial z}\right] = c(T)\,\gamma(T)\,\frac{\partial T}{\partial \tau} \tag{5}$$

can, by means of the integral transformation

$$\Phi = \frac{1}{\lambda_0}\int_0^T \lambda(T)\,dT \tag{6}$$

be reduced to

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = \frac{1}{a(T)}\cdot\frac{\partial\Phi}{\partial\tau}. \tag{7}$$

For many materials whose thermophysical properties depend strongly on the temperature, the thermal diffusivity $a(T) = \lambda(T)/c(T)\gamma(T)$ varies only slightly over the operating temperature range and may be averaged. Then Eq. (7) becomes linear and can be simulated by a conventional RC network. The error in the solution incurred by averaging the coefficient a will be insignificant or at least much smaller than when λ = const and $c\gamma$ = const. The reason for this is that the relation $\lambda = \lambda(T)$, as has been mentioned earlier, is accounted for in the simulation of the boundary conditions. When coefficient $a(T)$ cannot be averaged, however, then a special model is required [4] for simulating Eq. (7). But even then the device which will be described here becomes necessary for solving reverse problems.

The boundary condition of the third kind (2), with relation $\lambda = \lambda(T)$ taken into account, becomes after transformation according to (6):

$$\alpha(\tau)\,[T_m(\tau) - T_s(\Phi_s, \tau)] = -\lambda_0\left(\frac{\partial\Phi}{\partial n}\right)_s. \tag{8}$$

It is not possible to solve Eq. (8) with the aid of the devices which have been described earlier, since during simulation of Eq. (7) the potentials $U_\Phi(\tau)$ at the terminal points become proportional now not to the temperature $T_S(\Phi_S, \tau)$ but to the variable $\Phi(\tau)$. In order to convert voltage $U_\Phi(\tau)$ into voltage $U_r(\tau)$ in accordance with the functional relation $T = f(\Phi)$ based on (6) for a specified known $\lambda = \lambda(T)$ relation, one must connect a nonlinear converter (NC) into the feedback circuit of these devices, as is shown in Fig. 1c. Now it is possible to determine $q_S(\tau)$ and $\alpha(\tau)$, i.e., to solve a nonlinear reverse transient-heat-conduction problem with variable boundary conditions of the third kind.

For the synthesis of such devices, the passive networks must be augmented by all these components. It is to be noted that existing models such as the USM-1 [5], for example, contain these components except the MULT and the NC. But even they could be incorporated into a USM-1, based on DCA channels for the boundary conditions of the first kind, which would make it possible to solve here reverse boundary-value problems.

With the aid of these devices, several reverse transient-heat-conduction problems were solved on the USM-1 model. For illustration, we show in Table 1 the results of such a solution for the case of an asymmetrically heated infinitely large plate, grade EI-607 steel $l = 0.3$ thick, where the rate of heat transfer at the surfaces under transient conditions has been determined. As starting data we used those describing the variation of the dimensionless temperature, which had been obtained in [6] in the process of an analytical solution of the forward problem. According to this table, the discrepancy between the results of electrical simulation and of analytical calculations is on the average not greater than 1-2%, which indicates an agreement between both solutions.

Thus, a combination of passive networks and devices designed on the principle of electronic simulation makes it possible to considerably extend the applicability of existing models by adapting them for the solution of forward as well as reverse linear and nonlinear transient-heat-conduction problems with variable boundary conditions.

NOTATION

T	is the temperature, °C;
α	is the heat-transfer coefficient, $W/m^2 \cdot deg$;
λ_0	is the thermal conductivity λ at $T = 0$, $W/m \cdot deg$;
Φ	is the new variable, °C;
c	is the specific heat, $W/kg \cdot deg$;
γ	is the density of material, kg/m^3;
q_s	is the thermal flux density at the body surface, W/m^2;
τ	is the time, h;
U	is the voltage, V;
I	is the current, A;
K_t	is the conversion factor from T and Φ to U, V/deg;
K_R	is the conversion factor from thermal resistance to electrical resistance, $\Omega W/m^2 \cdot deg$;
K_α	is the conversion factor from α to U_α, $m^2 \cdot deg/A$;
K_T	is the conversion factor from q_s to I_T, m^2/V;
K	is the transfer ratio of the DCA (dc amplifier);
K_i	is the transfer coefficient of the CCS (controlled current stabilizer) channel, A/V;
K_a	is the transfer coefficient of the MULT (multiplicator), V^{-1};
$\theta_s = T_s/T_{m_2}$	is the dimensionless temperature;
$Fo = a\tau/l^2$	is the Fourier number;
$Bi = \alpha l/\lambda$	is the Biot number;
$Ki = ql/\lambda(T_{m_2} - T_0)$	is the Kirpichev number.

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